STABILITY EVALUATION FOR DIGITAL RADIOELECTRONIC DEVICES ABOVE 2ND ORDER

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The aim of the study is to simplify the digital radioelectornic devices above second order based on the stability triangle and inequations of the acceptable values of coefficients of transfer functions. The maximum permissible coefficient values are found, as well as the limiting inequations for the transfer function coefficients. The correlation coefficients between easily determined maximum theoretical coefficient values and their practical values are determined, functional dependences of correction coefficients are obtained.

Keywords: Jury criterion, characteristic equation, stability boundaries, limiting inequations, frequency-dependent component, robotic complexes.

The modern digital electronic systems, programmable mobile and robotic systems and complexes are functioning under complicated and unpredictable conditions, which implies the need to rearrange the transfer function coefficients of the system components. Therefore, we face a problem when, prior to start the adjustment, it is necessary to evaluate the system stability while ensuring its new condition, by changing the transfer function coefficients of the system components. For first and second order transfer functions, this problem is solved easily, but the problem becomes more complicated when dealing with transfer functions above the second order.

This research aimed to simplify the process of system stability assessment for the transfer functions above second order based on the stability triangle and inequations of allowable values of transfer function coefficients.

In most stability problems, the system characteristic equation is considered:

$$D(z) = \sum_{i=0}^{n} b_i z^{n-i},$$

where b_i is the denominator coefficients at the system transfer function (in most cases, $b_0 = 1$), n is the system order.

The n^{th} order system stability can be assessed using the Jury criterion, according to which the inequations of the characteristic equation of the following form will be satisfied

$$D(1) > 0;\ (-1)^n D(-1) > 0. \tag{1}$$

On order to assess the stability of tunable transfer functions above second order, it is necessary to solve the corresponding algebraic equations. However, we suggest the following procedure. Let us consider the following 5th order characteristic equation:

$$D(z) = z^5 + b_1 z^4 + b_2 z^3 + b_3 z^2 + b_4 z + b_5.$$

Then, according to the Juri criterion we can write

$$\begin{cases} 1 + b_1 + b_2 + b_3 + b_4 + b_5 > 0, \\ 1 - b_1 + b_2 - b_3 + b_4 - b_5 > 0. \end{cases}$$

We find that the lines forming the stability triangle will have the form

$$\begin{cases} 1 + d_1 + d_2 > 0, \\ 1 - d_1 + d_2 > 0 \end{cases} \text{ or } \begin{cases} d_2 > -1 - d_1, \\ d_2 > -1 + d_1 \end{cases}$$

In coordinates (d_1, d_2) we again do not observe the stability upper limit, which by analogy can be found from the Viet theorem and Euler formulas, taking into account the roots $z_0 = \alpha$, $z_{1,2} = \beta e^{\pm j\phi}, z_{3,4} = \gamma e^{\pm j\psi}$:

$$\begin{cases} \alpha + 2[\beta\cos\phi + \gamma\cos\psi] = -b_1, \\ \beta^2 + \gamma^2 + 2\alpha(\beta\cos\phi + \gamma\cos\psi) + 4\beta\gamma\cos\phi\cos\psi = b_2, \\ \alpha\beta^2 + \alpha\gamma^2 + 4\alpha\beta\gamma\cos\phi\cos\psi + 2\beta\gamma[\beta\cos\psi + \gamma\cos\phi] = -b_3, \\ 2\alpha\beta^2\gamma\cos\psi + 2\alpha\beta\gamma^2\cos\phi + \beta^2\gamma^2 = b_4, \\ \alpha\beta^2\gamma^2 = -b_5. \end{cases}$$

Based on these equations, we can find limiting in equations for both the coefficients and the sum of even and odd coefficients:

$$\begin{cases} -3 < -b_1 < 5, \\ -6 < b_2 < 10, \\ -2 < -b_3 < 10, \\ -3 < b_4 < 5, \\ 0 < b_5 < 1 \end{cases} \text{ and } \begin{cases} -9 < d_1 < 16, \\ -6 < d_2 < 15. \end{cases}$$

The resulting ratio allows generalization. Based on the analysis of the maximum values of characteristic equation coefficients in the worst case, we can specify that these values are determined by the combination formula

$$C_n^m = \frac{n!}{m!(n-m)!}, \tag{2}$$

where *n* is characteristic equation order (or the frequency-dependent component transfer function order), *m* is coefficient number, C_n^m is coefficient value (e.g., $b_3 = C_5^3$ for the fifth order equation coefficient).

Thus, it is possible to calculate the maximum values of the coefficients for all characteristic equation coefficients at any order of the transfer function.

Studies and analysis of the upper boundary of the stability triangle, determined by the sum of even coefficients, showed that it can be calculated by the relation

$$d_{2\max} = 2^{n-1} - 1,$$

where *n* is the characteristic equation order (or the frequency-dependent component transfer function order).

Having analyzed the transfer functions for computer system frequency-dependent components of orders above second, we revealed that the obtained estimates are higher than the possible values of the coefficients in the entire frequency range representing the stability triangles of theoretical calculations using formula (2) and their experimental verification. In this case, the theoretical assessment can be considered as the upper limit of the characteristic equation coefficients, which is overrated, but can be easily calculated.

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Оценка устойчивости цифровых радиоэлектронных устройств выше второго порядка

Целью настоящей работы является упрощение процесса оценки устойчивости радиоэлектронных устройств выше второго порядка на основе треугольника устойчивости и неравенств допустимых значений коэффициентов передаточных функций. Найдены максимально допустимые значения коэффициентов, а также ограничивающие неравенства для коэффициентов передаточной функции. Выявлены коэффициенты соответствия между легко определяемыми максимальными теоретическими значениями коэффициентов и практическими их значениями, получены функциональные зависимости корректирующих коэффициентов.

Ключевые слова: критерий Джури, характеристическое уравнение, границы устойчивости, ограничивающие неравенства, частотнозависимый компонент, робототехнические комплексы.